

B. TECH.**AM-101 : ENGG. MATHEMATICS - I**

- Q.1**
- (i) Find the general value of $\text{Log}(-2)$
- (ii) Find the Equation of Cone with vertex at origin and which passes through the curve
 $x^2+y^2=4, z=2$
- (iii) If $z = x^2y, x^2+xy+y^2 = 1$ find $\frac{dz}{dx}$
- (iv) Find the Equation of tangent plane to the surface $2x^2+y^2+2z = 3$ at the point $(2, 1, -3)$
- (v) Find the radius of curvature at the origin for the curve $y^2 = x^2 \left(\frac{a+x}{a-x} \right)$, a u any the Constant.
- (vi) Discuss the Convergence of $\sum_{n=1}^{\infty} \left(\frac{n+1}{3n} \right)^n$
- (vii) Find the entire length of the cardioid $r = a(1 - \cos\theta)$
- (viii) Change the order of integration $\int_0^a \int_y^a f(x,y) dx dy$
- (ix) Define Gamma function \sqrt{n} & prove that $\sqrt{(n+1)} = n\sqrt{(n)}$
- (x) If $x=r\cos\theta, y=r\sin\theta$ find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

PART - A

- Q.2** (a) Sketch the curve $y=x^3-6x^2+9x+6$.

- (b) Prove that radius of curvature at any point $p(x,y)$ on the Ellipse $x=acos\theta$, $y=bsin\theta$ is $\frac{a^2b^2}{p^3}$ where p is length of perpendicular from the centre of the Ellipse on the tangent at P.

Q.3 (a) If $u = \tan^{-1} \frac{y^2}{x}$

Show that, $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\sin 2u \sin^2 u$.

- (b) If Z is function of x and y and u, v be two other variables such that $u = lx+my$, $v = ly-mx$ Show that $\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2} = (l^2+m^2) \left(\frac{\partial^2 Z}{\partial u^2} + \frac{\partial^2 Z}{\partial v^2} \right)$.

Q.4 (a) Find Taylor series of $x^2-3y^2-9y-9x+26$ of maximum order at the point $(2, 2)$

- (b) Prove that of all rectangular parallelepiped with same volume, the cube is the least Surface.

Q.5 (a) Find the length of the arc of parabola $x^2=49y$ extending from the vertex to one extremity of the latus rectum.

- (b) Find the volume of the solid obtained by revolving the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about x axis.

PART B

Q.6 (a) Find the Equation of right circular cylinder of radius 2, whose axis lies along the line

$$\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5}$$

- (b) Find the Equation of sphere touching the circle $x^2+y^2+z^2+10y-4z-8=0$, $x+y+z=3$ as one of its great circles.

Q.7 (a) Find the area bounded by the parabola $y = x^2$ and the line $y = 2x+3$

- (b) Evaluate $\iint \sqrt{a^2-x^2-y^2} dx dy$ over the semicircle $x^2 + y^2 = ax$ in the positive quadrant by changing into polar coordinates.

Q.8 (a) Discuss the Convergence of series

$$1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \dots, x > 0$$

(b) Discuss convergence the tollouins series

$$(i) \sum_{n=1}^{\infty} (\sqrt{x^2+1}-n) \quad (ii) \sum_{n=1}^{\infty} \frac{(-1)^n}{x^2+1}$$

Q.9 (a) Sum the series :

$$\sin \alpha + x \sin(\alpha+\beta) + \frac{x^2}{2} \sin(\alpha+2\beta) + \dots$$

(b) Prove that roots of the Equation $(x-1)^n = x^n$

$$(n \text{ being a the integer}) \text{ are } \frac{1}{2} \left(1 + i \cot \frac{r\pi}{n} \right)$$

Where $r = 1, 2, \dots, n-1$.